Problems

**Fluids - 2**

This problem will exploit dimensional analysis to study the flow of a viscous fluid between two parallel plates. Consider the geometry depicted below.

Between the two plates, a two-dimensional incompressible flow occurs in the x-y plane. Let \( u \) and \( v \) refer to the velocity component in the \( x \) and \( y \) direction, respectively. The density, pressure, and kinematic viscosity are \( \rho \), \( p \), and \( \nu \) respectively.

1. **[30 points]** Define the characteristic flow velocities in the “x” and “y” directions as \( U \) and \( V \), respectively. Through extensive experimentation it has been discovered that \( V \) is independent of both the pressure and the fluid properties. Using dimensional analysis, define a set of \( \Pi \) groups that describe \( V \).

2. **[40 points]** Under the assumptions of Problem 1, the governing equation for the fluid velocity is,

\[
\frac{\partial^2 u}{\partial y^2} = \Psi
\]

\[
\frac{\partial^2 v}{\partial y^2} = 0
\]

where \( \Psi \) is a constant related to the pressure gradient. Solve for the velocities subject to the boundary conditions:

\[
u(x, 0) = 0
\]

\[
u(x, h) = U
\]

\[
u(x, 0) = 0
\]

\[
u(x, h) = 0
\]

3. **[30 points]** Consider the case, \( U = 0 \), and denote the areal flow (volumetric flow rate per unit depth) between the plates as \( Q \). Find an expression for \( Q \) in terms of \( \Psi \) and the geometric variables \( h \), and \( L \).