Problems

Consider the following unity negative feedback system

\[ K=1, \quad G(s)=\frac{9}{s(s+a)} \]

1) Find the closed-loop transfer function \( G_{CL}(s) = \frac{Y(s)}{U(s)} \) \( (8 \text{ points}) \)

2) Calculate the undamped natural frequency \( (\omega_n) \) of the closed-loop system. \( (8 \text{ points}) \)

3) Find a value for “\( a \)” such that the closed-loop system damping ratio \( (\zeta) \) is 20%. \( (8 \text{ points}) \)

4) Find the 2% settling time and the percent overshoot of the closed-loop system response due to a step input \( (8 \text{ points}) \)

5) Plot the root locus of the system by varying the gain \( K \). \( (8 \text{ points}) \)

6) On the root locus, mark the poles that correspond to critical damping. \( (7 \text{ points}) \)

7) What is the value of gain \( K \) to achieve the critical damping? \( (8 \text{ points}) \)

Let us consider a new plant \( G(s) \), the Bode plot is in the next page.

8) Which one of the following transfer functions represent \( G(s) \), why? \( (8 \text{ points}) \)

   (a) \( G(s)=\frac{s}{(s+a)} \),  (b) \( G(s)=\frac{1}{s(s+a)} \),  (c) \( G(s)=\frac{s}{s^2+as+b} \),  (d) \( G(s)=\frac{1}{s^2+as+b} \)

9) From the Bode plot, assuming that \( G(s) \) is the open loop transfer function, what is the gain margin? \( (7 \text{ points}) \)

10) Again, assuming that \( G(s) \) is the open loop transfer function, what is the phase margin? \( (7 \text{ points}) \)

11) If a unity negative feedback loop were closed around \( G(s) \), would the system be stable at gain \( K=1? \) \( (7 \text{ points}) \)

12) On a separate figure, sketch the Nyquist plot of \( G(s) \) using data from the Bode plot. \( (8 \text{ points}) \)

13) Clearly mark the gain margin and phase margin on the Nyquist plot. \( (8 \text{ points}) \)
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