Vibrations - 1

Question 1: (10 points)
- Can you use superposition principle to solve a non-linear differential equation?
  YES    NO    (circle your answer)
- Write an example of a non-linear differential equation?

Question 2: (15 points)
- Find magnitude and phase of the following complex number \( z = -4e^{i\pi/4} \)
- Sketch the complex number \( z \) in the complex plane

Question 3: (15 points)
Find spring deformation, \( e \), in terms of \( x \) and \( \theta \) assuming small displacements, i.e. change in angle, \( \Delta \theta \), is much smaller than \( \theta \).

Fall 2019
Question 4: (10 points)
The free response of a 2nd order mechanical system for two damping ratios (110% and 140%) is shown in the figure. Indicate in the figure the response corresponding to 140% damping ratio.

Question 5: (15 points)
The mechanical system is allowed to vibrate freely from the static equilibrium position. The time history of the displacement is shown below. If the mass of the system is $100 \text{ N.s}^2/\text{m}$, estimate:

<table>
<thead>
<tr>
<th>Initial displacement $x_0$:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial velocity $x_v$:</td>
<td></td>
</tr>
<tr>
<td>the natural frequency $\omega_n$:</td>
<td></td>
</tr>
</tbody>
</table>

The free response of a 2nd order mechanical system for two damping ratios (110% and 140%) is shown in the figure. Indicate in the figure the response corresponding to 140% damping ratio.
Problems

Question 6: (10 points)
What is the particular solution of \( m_{eq} \ddot{x}_p + b_{eq} \dot{x}_p + k_{eq} x_p = P e^{j\omega t} \)?

What is the particular solution of \( m_{eq} \ddot{x}_p + b_{eq} \dot{x}_p + k_{eq} x_p = P \cos(\omega t) \)?

NOTE: Please write the expression for the particular solution. NO need to solve for any unknown parameters in the solution.

Question 7: (15 points)
Estimate the amplitude of the steady-state response \( x(t) \)

\[
\dot{x}(t) + x(t) = \sin(10t)
\]

Question 8: (10 points)
Estimate the frequency in Hz of the first 3 harmonics for the following periodic signal.