Problems

Mathematics - 1

Part a (50 points)

1. (10 points) Let \( g(x) = ax^2 + bx + c \) what is \( \frac{dg(x)}{dx} \)?

2. (20 points) Let \( F(y) = \frac{1}{\sqrt{y}} \) what is \( \frac{dF(y)}{dy} \)?

3. (20 points) Substituting \( g(x) \) into \( y \), what is \( \frac{dF(x)}{dx} \)?

Part b (50 points)

For two point bodies \( i \) and \( j \), let \( \mathbf{r}_i \) and \( \mathbf{r}_j \) be position coordinate vectors in 3D:

\[
\mathbf{r}_i = \begin{bmatrix}
r_{ix} \\
r_{iy} \\
r_{iz}
\end{bmatrix} \quad \mathbf{r}_j = \begin{bmatrix}
r_{jx} \\
r_{jy} \\
r_{jz}
\end{bmatrix}
\] (2)

The Euclidean norm between the bodies is given by:

\[
\| \mathbf{r}_i - \mathbf{r}_j \| = \sqrt{(r_{ix} - r_{jx})^2 + (r_{iy} - r_{jy})^2 + (r_{iz} - r_{jz})^2}
\] (3)

1. (20 points) Expand the Euclidean norm into the form \( \sqrt{ax^2 + bx + c} \) and substitute \( r_{ix} \) for \( x \). What are \( a; b; c \) (5 points each) in terms of the remaining elements of \( \mathbf{r}_i \) and \( \mathbf{r}_j \)?

2. (30 points) Let \( F(\mathbf{r}_i, \mathbf{r}_j) = \frac{1}{\| \mathbf{r}_i - \mathbf{r}_j \|} \). Show that:

\[
\frac{dF(\mathbf{r}_i, \mathbf{r}_j)}{dr_{ix}} = -\frac{(r_{ix} - r_{jx})}{\| \mathbf{r}_i - \mathbf{r}_j \|^3}
\] (4)