Vibrations - 1

Question 1: (10 points)
- Can you use superposition principle to solve a non-linear differential equation?
  YES  NO  (circle your answer)
- Write an example of a non-linear differential equation?

Question 2: (15 points)
- Find magnitude and phase of the following complex number $z = -4e^{i\pi/4}$
- Sketch the complex number $z$ in the complex plane

Question 3: (15 points)
Find spring deformation, $e$, in terms of $x$ and $\theta$ assuming small displacements, i.e. change in angle, $\Delta \theta$, is much smaller than $\theta$. 
**Question 4: (10 points)**
The free response of a 2\textsuperscript{nd} order mechanical system for two damping ratios (110\% and 140\%) is shown in the figure. Indicate in the figure the response corresponding to 140\% damping ratio.

**Question 5: (15 points)**
The mechanical system is allowed to vibrate freely from the static equilibrium position. The time history of the displacement is shown below. If the mass of the system is 100 N.s\textsuperscript{2}/m, estimate:

- Initial displacement $x_0$:
- Initial velocity $\dot{x}_0$:
- the natural frequency $\omega_n$:

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Question 6: (10 points)
What is the particular solution of \( m_{eq} \ddot{x}_p + b_{eq} \dot{x}_p + k_{eq} x_p = P e^{i\omega t} \)?

What is the particular solution of \( m_{eq} \ddot{x}_p + b_{eq} \dot{x}_p + k_{eq} x_p = P \cos(\omega t) \)?

NOTE: Please write the expression for the particular solution. NO need to solve for any unknown parameters in the solution.

Question 7: (15 points)
Estimate the amplitude of the steady-state response \( x(t) \)

\[ \dot{x}(t) + x(t) = \sin(10t) \]

Question 8: (10 points)
Estimate the frequency in Hz of the first 3 harmonics for the following periodic signal.