Problems

A clamped beam with circular cross section, as shown in the figure, is subjected to the axial force \( P \), transverse force \( F \), and Torque \( T \). The diameter and length are 15 mm and 100 mm, respectively. Consider the following load cases and complete the assignments in each case:

**Case I:** \( P=0 \), \( F=0 \), and \( T \neq 0 \)

a. Consider two identical beams made of aluminum and steel subjected to this load case (the same torque is applied to both beams and \( P=F=0 \)). Which beam will have the higher shear stress (5 Points)? Which beam will have a larger angle of twist (5 Points)? Support your answers with appropriate equations. Note that the Modulus of Rigidity (\( G \)) for steel is almost 3 times higher than aluminum.

b. Now, assume the beam is made of a ductile material with yield strength \( S_y \). Using the Maximum Shear Stress Theory (10 Points) and Distortion Energy Theory (10 Points) find the maximum Torque which can be applied to the beam as a function of \( S_y \).

**Case II:** \( P=4 \) KN, \( F=0.55 \) KN, \( T=25 \) N.m

c. Assume that the transverse shear stresses are negligible. Identify the critical stress element (point A or B on figure) and find the nonzero components of stresses at that point (10 Points). Hint: You can find the critical element without any calculations and by looking only at the directions of nonzero stress components at each element.

d. Construct a Mohr's circle for the stress element at the critical points and find the principal stresses and maximum shear stress at that point (20 Points)

e. Assume the beam is made of a ductile material with \( S_y =280 \) MPa. Using the Maximum Shear Stress (MSS) Theory, determine the factor of safety for the critical element (20 Points).

f. Plot the Maximum Shear Stress theory and the Distortion Energy Envelopes and show the load line on the figure (10 Points). If the Distortion Energy theory was used in part (e), do you expect the calculated factor of safety to be smaller or larger (10 Points)?