Controls - 2

Consider the block diagram shown below.

In this unity feedback control system the commanded input is $r$, the output is $y$, $d$ and $n$ represent injected noise, $C(s)$ represents a compensator to be designed, and $P(s)$ represents the plant.

1.) **(15 points)** Derive the transfer function matrix from the input(s) to the output(s). Define the vector of injected noise $N(s) := [n(s), d(s)]^T$. What are the sensitivities and the complementary sensitivities for this system in terms of $r(s)$ and $N(s)$?

2.) **(15 points)** The plant $P(s)$ has the block diagram below.

Derive the closed loop transfer function for the plant $P(s)$.

3.) **(20 points)** Now suppose for all the parts below that $d(s)=n(s)=0$, $C(s)=k$, and $P(s) = \frac{s^2 + 21s + 110}{s^2 - 2s + 2}$. Sketch the root locus for this system. Be sure to include breakaway points, angle of departures, or angle of arrivals if there are any for this system.

4.) **(10 points)** Use Routh-Hurwitz criterion to find the critical value of the gain $k$ as a pole crosses the imaginary axis.

5.) **(10 points)** Use the magnitude criterion and your results from part (4.) to solve for the location of the closed loop pole location as it crosses the imaginary axis.

6.) **(15 points)** Use the final value theorem to find the steady state error of the closed loop system to a unit step input.
7.) (15 points) Explain your answer to part (6) using type number analysis. If we change the controller to the integrator $C(s) = \frac{k}{s^2}$, use type number analysis to find the steady state error of the closed loop system to a unit ramp input.