Ph.D. QUALIFYING EXAM – Spring 2019
Description of Areas to be Examined

The fundamental principles of mathematics and the six basic areas of mechanical engineering to be covered by the examination and the minimum skills expected of the examinees in these areas are outlined below. While the list below is a fairly comprehensive guide, the exam may include any techniques found in a typical undergraduate Mechanical Engineering curriculum.

I. Mathematics

Suggested Texts:

1. Wylie, C.R. and Barrett, L. *Advanced Engineering Mathematics*, 6th Ed., McGraw-Hill, 1995
2. Boyce, W.E. and DiPrima, R.C. *Elementary Differential Equations and Boundary Value Problems*, 6th Ed., John Wiley, 1996
3. Chapra, C.C and Canale, R.P. *Numerical Methods for Engineers*, 4th Ed., McGraw-Hill, 2002

A. Fundamental Principles and Skills

1. **Geometry and Trigonometry**: Be able to apply plane geometry and trigonometry to determine required dimensions of physical systems. No proofs.
2. **Algebra**: Be able to write systems of equations in matrix form, solve linear simultaneous equations, find roots of polynomial expressions. Both analytical techniques and graphical visualization are required. Be familiar with Fourier series and Taylor series, the Dirac delta function, step functions, probability density functions.
3. **Differentiation and Integration**: Be able to perform the integration and differentiation required in the formulation and solution of physical problems. Both analytical techniques and graphical visualization are required. Be familiar with convolution integrals, power series.
4. **Ordinary Differential Equations**: Be able to solve first and second order ordinary differential equations subjected to a variety of inputs and initial conditions. Both analytical techniques and graphical visualization are required. Be familiar with a variety of solution techniques including Laplace Transforms, separation of variables, and others.
5. **Numerical Methods**: Be able to apply numerical methods (e.g. Euler's Method, trapezoidal method, Newton-Raphson method, etc.) to the solution of algebraic equations, to integration, to differentiation, and to the solution of first order differential equations. Be able to construct algorithms for applying these numerical methods.
6. **Linear Algebra:** Be familiar with matrix operations, including finding the transposes, determinants, inverses of matrices. Find eigenvalues and eigenvectors, derive least-squares solutions.

II. Dynamics and Vibrations

Suggested Texts:

1. James, *Vibration of Mechanical and Structural Systems*, Harper, 1989
2. Rao, *Mechanical Vibrations*, Addison Wesley, 2nd ed., 1990
3. Inman, D. J., *Engineering Vibration*, 3rd ed., Prentice Hall, 2007

A. Fundamentals Principles and Skills

1. **Modeling.** Be able to develop the equations of motion that describe mechanical systems in translation and/or rotation. Requires knowledge of the basic system elements of mass, damper, and spring. Models are usually derived from free-body diagrams, but may also be developed from energy methods.
2. **Response.** The student must be able to solve ordinary differential equations and compute the free response or total response, but much design work requires the solution for the steady state forced response to harmonic excitation. Solution of periodic and non-periodic excitation are also covered in this course and used for design work.
3. **Application.** Some application areas relevant to vibrations include the design of isolation mounts and force transmitted to the support structure, application of log decrement to solve for equivalent system damping, determination of natural frequency and mode shapes for two degree of freedom systems, and the design of vibration absorbers. Systems with constant force excitation and excitation from rotating imbalance are very common.

B. Fundamental Mathematical Skills

1. Solution of Ordinary Differential Equations. For first- and second-order systems classical methods of solutions are employed for simple inputs.
2. Solution of Algebraic Equations. Various root-finding methods are employed to determine the system eigenvalues. Typically this involves finding the roots of polynomial expressions.

C. Sample Undergraduate Syllabus (Based on Inman, D. J., *Engineering Vibration*, 3rd ed., Prentice Hall, 2007. Does not include class handouts and other non-text materials.)

Topic	Reading	Problems
SDOF, free vib, harm motion, undet coeff	1.1, 1.2, 1.3	1.2(a), 1.7
Units, SDOF sol methods	1.1, 1.2, 1.3	1.11, 1.14, 1.31, 1.40
Modeling and energy methods	1.4	1.52a,b 1.54 a,b
Equiv.stiffness, equiv. mass, D'Alembert	1.5	1.59, 1.66, 1.68, 1.77
Measurement, log dec.; design	1.6	1.78, 1.81, 1.82, 1.85
Stability; Demo of instability	1.8	1.43, 1.92
Numerical Simulation	1.9, 1.10	1.95, 1.104, 1.106
Forced response of SDOF systems	2.1, 2.2	2.7, 2.26, 2.21, 2.22
Forced response of SDOF systems	2.2, 2.3.2	2.30, 2.27, 2.37
Base motion, transmissibility	2.4	
Rotating imbalance, demo	2.5	2.51, 2.53, 2.56
Impulse response function, convolution	3.1	3.5, 3.6, 3.8
Response to an arbitrary input	3.2	3.18
Response to periodic inputs	3.3	3.31
Shock Spectrum, discrete Fourier,	3.6, 3.9, 3.10	3.43
Shock Spectrum	3.3	
Vibration isolation	5.1, 5.2, 5.9	5.6, 5.8, 5.20
2DOF models, solution, eigenvalues	4.1, 4.2	4.1, 4.2, 4.3
Mode shapes	4.2, 4.5	4.4, 4.11
Forced response of damped 2DOF systems	4.5, 4.6	
Vibration absorbers	5.1, 5.3, 5.4	5.30, 5.34
multi DOF		

III. Machine Design

Suggested Texts:

1. R. Shigley, and L. Mitchell, *Mechanical Engineering Design*, 4th ed., McGraw-Hill, 1988.
2. R. Shigley, and C. Mischkey, *Mechanical Engineering Design*, 5^{th} or 6^{th} ed., McGraw-Hill.
3. Juvinall and Marshek, *Fundamentals of Machine Component Design*, 2nd ed., John Wiley, 1991.
4. R. G.Budynas and J. Keith Nisbett *Mechanical Engineering Design*, 8th ed., McGraw-Hill, 2007.

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A. Fundamental Principles and Skills

1. **Statics**. Be able to construct free-body diagrams and determine equilibrium forces and moments overall and on internal sections for stress calculations.
2. **Static Failure Theory**. Be able to design components with combined stress states for a prescribed or selected factor of safety or analyze existing components.
3. **Fatigue Failure Theory**. Be able to design or analyze components for fatigue failure.
4. **Stress Analysis**. Be able to calculate stresses in simple components like trusses, beams, thin-wall pressure vessels, shafts, etc.
5. **Design Practice**. Simplifying assumptions, stress concentration factors, factor of safety, common material behavior.

B. Fundamental Mathematical Skills

1. **Differentiations and Integration**. Be able to perform the integrations and differentiation as required to derive expressions for the shear-load, bending-moment, slope and/or deflections of statically determinate beams and solve the resulting boundary-value problems. Analytical, numerical and graphical skills are required.
2. **Differential Equations**. Be able to solve ordinary differential equations as are used, for example, in buckling analysis.
3. **Vector Analysis**. Be able to perform basic vector mathematics to determine magnitudes and orientation of loads and deflections.
4. **Algebraic Equations**. Be able to find the roots of polynomial expressions, write linear equations in matrix form, and solve linear simultaneous equations.

C. **Sample Undergraduate Syllabus (**Based on R.G. Budynas and J. Keith Nisbett *Mechanical Engineering Design*, 10th ed., McGraw-Hill, 2014. Does not include class handouts and other non-text materials.)

Topic	Reading	Suggested problems
Design, safety factor Materials: properties, processing, and selection	1-1 thru 1-3, 1-7 thru 1-12 2-1, 2-7, 2-12, 2-13, 2-15	 2-1, 2-5
Equilibrium, FBD, shear and bending diagram Stress, strain, principal stresses, combined loading	3-1 thru 3-3 3-4 thru 3-8	3-8 3-15 (a and b), 3-20
Normal stress (axial, bending) Shear stress (transverse shear, torsion)	3-9 thru 3-10 3-11 thru 3-12	3-44 3-64
Combined stresses (2-plane bending), stress concentration Pressure vessels, press and shrink fits, contact stresses	3-12 thru 3-13 3-14, 3-16, 3-19	3-84 3-92, 3-96
Static failure theory: ductile (MSS)	5-1 thru 5-4	5.1 (a and e), 5.3 (a and e)
Static failure theory: ductile (DE, CM) Static failure theory: brittle (MNS, BCM, MM)	5-5 thru 5-7 5-8 thru 5-10	5.1 (c and d), 5.10, 5.12 5.21, 5.25
Buckling: Euler, Johnson; buckling design FEA introduction and fracture Introduction	4-11 thru 4-13 5-12	4-104, 4-105, 4-106 5-84, 5-85
Fatigue introduction Fatigue strength estimation	6-1 thru 6-4 6-7 thru 6-8	6-5 6-3
Fatigue stress concentration Fluctuating stresses, fatigue failure theory	6-9 thru 6-10 6-11 thru 6-12	6-16 6-27(a)
Fluctuating torsion, combined loading Overload introduction and formulation	6-13 thru 6-14 6-16 thru 6-17	6-3
Cumulative fatigue	16-5	6-59, 6-60
Thread standards, power screws, stiffness model Bolt strength, preload	8-1 thru 8-5 8-6 thru 8-8	8-1, 8-4, 8-11 8-26
Bolted joint in tension static design Bolted joint in tension fatigue design, shear design	8-9 thru 8-10 8-11 thru 8-12	
Welds introduction, butt and fillet welds Welded joint stress: torsion, bending	9-1 thru 9-2 9-3 thru 9-4	
Weld strength	9-5 thru 9-7	

IV. Control Theory

Suggested Texts:
1. Dorf and Bishop, *Modern Control Systems*, 11th Ed., Prentice-Hall, 2008.
2. Ogata, *Modern Control Engineering*, 5th Ed., Prentice-Hall, 2009

Any textbook that presents an introduction into the analysis and design of control systems is a suitable study guide.

A. Fundamental Principles and Skills

1. **Modelin**g. Be able to develop up to 5th order ordinary differential equation models for simple mechanical, electrical, and electro-mechanical systems.
2. **Open and Closed Loop System Analysis**. Be able to determine stability and characteristics of the response of open and closed loop systems using Routh-Huritz, Bode, or Root-Locus techniques. Be able to calculate gain and phase margins. For second-order systems, be able to determine settling time, rise time, percent overshoot, damping ratio, and natural frequency.
3. **Control System Design**. Be able to develop controllers to achieve performance specifications. Be able to design PID and lead-lag controllers to meet performance specifications.

B. Fundamental Mathematical Skills

1. **Laplace Transforms**. Be able to convert an ordinary differential equation from the time domain to the Laplace domain. Be able to convert equations from the Laplace domain to the time domain.
2. **Time Response**. Be able to develop an expression for the response of an open loop or closed loop system in the time domain.
3. **Root Locus and Bode Plots**. Be able to make Root Locus and Bode Plot sketches.
4. **Complex analysis**. Be able to manipulate transfer functions to obtain expressions for their magnitude and phase.

C. Sample Undergraduate Syllabus (Based on Dorf and Bishop, *Modern Control Systems*, 11th Ed., Prentice Hall, 2008. Does not include class handouts and other non-text materials.)

Topic	Reading	Problems
Introduction to Course and Overview Linearization and ODE's	Preface, 1.1 -1.12 2.1-2.3	
Laplace, pole-zeros Transfer funct. and block diagrams Block Dia. and Examples:	2.4 2.5-2.6 2.6	P1.15, E2.2, E2,17, P2.3, P2.9 and P2.51
MATLAB, Simulation Final Value, Performance, sensitivity Open-loop vs closed loop, Error	2.6, 2.10 4.1 -4.2 4.3, 4.5	E2.23, P2.51 (in matlab), MP2.6, E4.1 and E4.7 (a&b)
Disturbance Performance of 2nd Order sys. Performance of 2nd Order sys.	4.4 5.1, 5.3 5.3	E4.7 (c&d), E4.9, MP4.3, P5.4 and AP5.4
Test Signals, Extra pole/zero	5.2, 5.4, 5.6	P5.2 and AP5.5
Steady State Error Stability, Root locus	5.7 -5.14 6.1, 7.1 -7.2	P6.7 and MP6.2
Root Locus Root Locus Root Locus	7.3 7.3 7.3	E7.7, E7.19, P7.1, P7.6 and AP7.9
PID Controllers: DP Given PID Controllers PID Controllers	7.7 7.7 7.8 -7.12	P7.1 for K<0, P.7.19, DP7.13, MP7.1, MP7.6
Frequency Response Frequency Response Bode Plots	8.1-8.2 8.2 8.3	E 8.3 and P 8.2 a,b,c
Bode Plots Bode Plots Bode Plots	8.3	P8.2 and P8.23
Measurement and Specs Controllers Gain and Phase margins	8.4, 8.5 10.16	P8.15, P8.24, AP 8.2, MP8.7 and MP8.9
Nyquist Criterion	9.3, 9.4, 9.10	
Nyquist Criterion Lead Controllers	9.3, 9.4, 9.10 10.1 – 10.3	P9.2, P9.4, P9.16 and AP9.4
Lead Compensator Design Lag Control Lead-Lag Control	10.4 – 10.6 10.7-10.8 10.9-10.16	SD1a and SD1b

V. Heat Transfer

Suggested Text: 1. Bergman and Lavine., *Fund. of Heat and Mass Transfer*, 8th ed., John Wiley, 2017.

A. Fundamental Principles and Skills

1. **Energy Balance**. Be able to derive the "energy equation" on the basis of an energy balance on both finite and differential volume elements (solid or fluid). The derivation should include the possibility of internal heat generation (sources). Be able to obtain analytical solutions for the case of steady conditions in solids and liquids with convection conduction and radiation..
2. **Extended Surfaces**. Be able to derive the ordinary differential equation governing the temperature distribution in an extended surface (fin) and, for the case of a constant cross-section, be able to solve it subject to classical boundary conditions (insulated tip, infinitely long, known tip temperature).
3. **Lumped Heat Capacity**. Know and be able to apply the criterion for using the lumped heat capacity approach to solving transient problems, and be able to formulate and solve multi-domain problems using this approach.
4. **Radiation Heat Transfer**. Be able to set up and solve the equations governing radiation heat transfer among diffuse, gray surfaces. This includes the ability to evaluate the needed radiation view factors for simple geometries.
5. **Convection Heat Transfer**. Be familiar with the origins of and the role played by non-dimensional groups in local and average convection correlations. Be able to solve for heat transfer for constant-property flow across bluff bodies and for fully-developed internal flow heat transfer using correlations available in typical undergraduate texts.
6. **Conduction Heat Transfer**. Be able to derive and know how to solve the explicit form of the finite-difference equations for unsteady, multidimensional conduction in rectangular coordinates, and be able to discuss system stability.

B. Fundamental Mathematical Skills

1. **Ordinary Differential Equations**. Be able to solve ordinary, homogeneous and nonhomogeneous, second-order differential equations having constant coefficients subject to specified boundary conditions.
2. **Systems of Differential Equations**. Be able to solve coupled systems of ordinary, first-order differential equations having constant coefficients subject to specified initial conditions.
3. **Algebraic Equations**. Be able to use matrix methods to solve linear systems of algebraic equations.

C. **Sample Undergraduate Syllabus** (Based on Bergman and Lavine, *Fund. of Heat and Mass Transfer*, 8th ed., John Wiley, 2017. Does not include class handouts and other non-text materials.)

Period	**Topic**	**Reading Assignment**	**Problems Due**
1	Control Volumes		

2	Energy balances	1.1-1.7, 2.1-2.5	1.12, 1.21, 1.35
3	Plane and Radial geometry	3.1-3.4	1.51, 1.61, 2.6
4	Internal generation	3.5	3.5ab, 3.7, 3.49
5	Series/Parallel Paths		3.70, 3.77, S1Lab
6	Fins	3.6	3.11, 3.48ab, 3.65
7	Fin Arrays	4.1, 4.3	3.102, 3.107
8	Fins		3.111, S2Lab
9	Finite-difference method	4.4-4.6	3.105, 3.121, 3.124
10	Transient, Lumped capacity method	5.1-5.3	4.35, 4.42
11	Solution methods		4.45, 5.9, S3Lab
12	Transient solutions	5.3	4.52, 4.56a, 5.10
13	Semi-infinite solid	5.7	5.17, 5.19, 5.67
14	**Test Review**		5.25, 5.72a, S4Lab
15	**Test 1**		
16	External convection	6.1-6.3, 6.6-6.8	
17	Boundary Layers		6.5, 6.24
18	External convection	7.1-7.4	6.7, 6.8, 6.40
19	External convection	7.5, 7.9	7.13, 7.19
20	Integrating for average h		7.21, 7.34a, S5Lab
21	Internal flow basics	8.1-8.2	7.8, 7.55
22	Internal flow basics	8.3-8.5	7.32, 8.3, 8.13
23	Enthalpy balances		8.7a, 8.21, S6Lab
24	Internal flow applications	8.6, 8.10	7.65ab, 8.28a, 8.43a
25	Internal flow applications		8.30a, 8.27, 8.57
26	**h estimation**		8.64, S7
27			8.54, 8.50ab, 8.70
28			
29	Test Review		
30	**Test 2**		
31	Intro to heat exchangers – LMTD	11.1-11.3	
32	LMTD and MTD		11.2a, 11.15
33	Effectiveness-NTU analysis	11.4	11.1, 11.20, 11.23ab,
34	Effectiveness-NTU analysis	11.5, 11.7	11.11, 11.49, 11.50
35	Effectiveness-NTU analysis		11.12, 11.34, S9
36	Radiation shape factors	13.1	11.62, 11.63a, 11.64
37	Radiation exchange – black surfaces	13.2	12.17, 12.13a, 13.1
38	Energy balances		13.3, 13.15, S10
39	Radiation exchange – gray surfaces	13.3	13.4ab, 13.8, 13.17
40	Radiation exchange - enclosures	13.3	13.33, 13.25, 13.38
41	**Gray surfaces**		13.42abc, 13.43, S11Lab
42	Multimode Exchange	13.4	13.62ab, S12
43	Radiation problem review		13.63, 13.73ab, 13.77ab

VI. Fluid Mechanics

Suggested Text:

1. Munson, Young, and Okishi, *Fundamentals of Fluid Mechanics*, 5th ed., John Wiley and Sons, 2006.
2. R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics,* 4th ed., Wiley & Sons, 1992.
3. F. M. White, *Fluid Mechanics*, 3nd ed., McGraw-Hill, 1994.

A. Fundamental Principles and Skills

1. **Hydrostatics**. Be able to compute pressures within static fluid systems and force distributions on bounding surfaces.
2. **Conservation Principles**. Be able to apply the principles of conservation of mass, momentum, and energy to differential and finite volume elements to obtain the equations of continuity and motion.
3. **Incompressible Ideal Fluids**. Be able to solve one-dimensional steady flow (hydrodynamics) problems using Euler's and Bernoulli's equations.
4. **Incompressible Viscous Flow**. Be able to describe and discuss the concepts of laminar and turbulent boundary layers and the criterion for transition to turbulence in exterior and interior flows. Be able to use simple boundary layer models to compute frictional drag. Also, be able to compute the pressure drop for fully-developed laminar flow in pipes. Finally, be able to use drag coefficients and friction factors to compute drag and pressure drop for flow over bluff bodies and within pipes.

B Fundamental Mathematical Skills

1. **Solution of Algebraic Equations**. Be able to solve an implicit and/or transcendental equation by some numerical technique.
2. **Differential Equations**. Be able to solve ordinary differential equations such as occur in transient flow analysis and simple boundary layer problems.

C. **Sample Undergraduate Syllabus** (Based on Munson, Young, and Okishi, *Fundamentals of Fluid Mechanics*, 5th ed., John Wiley and Sons, 2006. Does not include class handouts and other non-text materials.)

Topic	Reading	Problems
Introduction	1	1.6, 23
Fluid Statics Fluid Statics	2.1-2.6, 2.11 2.7-2.8	1.29, 56, 62, 2.5, 2.6 2.25,31,38
Fluid Statics Transport Theorem	2.102.11 4.3 , 4.4	2.49, 52, 57 2.70, 4.56, 4.58
Continuity Momentum Equation	5.1-5.1.2 5.2-5.2.2	5.5, 7, 9 5.15, 19, 30
Momentum Equation Energy Equation	5.2-5.2.2 Handout (See Appendix A)	
Fluid Kinematics Bernoulli Equation	4.1-4.2.3,6.2.1 3.1-3.4	5.32, 34, 58, 59
Bernoulli Equation Dimensional Analysis	3.5 – 3.6 7.1-7.5	5.43, 3.19, 3.43, 4.1, 4.14 3.27, 57, 60, 5.52
Dimensional Analysis Dimensional Analysis	7.6-7.8 7.8-7.9	3.67 5.62, 7.1, 2, 4, 9
Internal Flow Internal Flow	8.1, 8.2 8.3.1, 8.4.1	7.21 a, b, 7.50, 56, 65
Internal Flow Internal Flow	8.4.1-8.4.3 8.5, 8.6	8.4, 7, 11, 5.108 5.121, 8.16, 29
External Flow External Flow	9.1-9.2.1 9.2.3-9.2.5	8.35, 36, 65 8.68, 76, 89
External Flow External Flow	9.2.6-9.3 9.4	8.59, 62, 73 9.2, 7, 9
Boundary Layer	Handout (See Appendix A)	9.46, 59, 87, 88
Differential Operator Navier-Stokes Equation	Handout (See Appendix A) Handout (See Appendix A)	9.23, 30, 62 9.29
Review		9.16, 22, 24

VII. Thermodynamics

Suggested Text:

1. Van Wylen and Sonntag, *Fundamentals of Classical Thermodynamics*, English / SI Version 3rd ed., John Wiley, 1986.
2. Cengel and Boles, *Thermodynamics – an Engineering Approach*, 5th ed, McGraw-Hill, 2006.
3. Moran and Shapiro, *Fundamentals of Engineering Thermodynamics*, 5th ed., Wiley, 2004.

A. Fundamental Principles and Skills

1. **The First Law**. Be able to apply the first law of thermodynamics to various closed systems and open systems for both steady flow and transient cases. Be able to handle situations involving ideal gases, ideal gas mixtures, solids, liquids, phase change, and/or chemical reactions.
2. **The Second Law**. Be able to apply the second law of thermodynamics various closed systems and open systems for both steady flow and transient cases. Be able to handle situations involving ideal gases, ideal gas mixtures, solids, liquids, phase change, and/or chemical reactions. Be able to calculate entropy changes and to determine whether specified processes are reversible, irreversible, or even possible. Be able to use the second law to determine the states, processes, and energy quantities that correspond to reversible devices.
3. **Physical Property Relationships**. Be able to either calculate or determine from tables of thermodynamic properties the quantities needed to work with the first and second Laws. This includes knowing how to work with ideal gases, mixtures of ideal gases, liquids, solids, and substances which change phase.

B. Fundamental Mathematical Skills

1. **Differentiations and Integration**. Be able to perform differentiation and integration as used in property definitions and the evaluation of path functions and property changes. Be able to evaluate partial derivatives as used in property definitions and relationships among properties.
2. **Differential Equations**. Be able to solve ordinary differential equations as used, for example, in transient flow analyses and property relationships.

C. **Sample Undergraduate Syllabus** (Based on Moran and Shapiro, *Fundamentals of Engineering Thermodynamics*, 5th ed., Wiley, 2004. Does not include class handouts and other non-text materials.)

Topic	Reading	Problems
Introd. and Basic Concepts		
Energy, Work, & Heat	1.1-1.7, 2.1-2.4	
1st Law, Closed Sys. States, P-V-T Relations	2.5, 3.1, 3.2	1.13, 1.35, 1.41
Retrieving Properties	3.3	2.7, 2.25, 2.31
Properties continued Compressibility, Ideal Gas	 3.4, 3.5	2.54, 2.58, 2.69, ex 3 & 5 (p. 122), 3.2
Specific Heats	3.3.5, 3.6, 3.7, 3.8	1.29, 3.7, 3.17, 3.34
Control Volumes Control Vol., Enthalpy	4.1, 4.2	3.42, 3.61, 3.74, 3.81, 3.84
Steady-Flow Systems	4.3	3.96, 3.101, 3.105
Steady-Flow Systems Transient-Flow Systems	4.4	4.4, 4.9, 4.10, 4.15, 4.18
2nd Law	5.1-5.3	4.24, 4.37, 4.44
Max. Perf., Carnot Cycle	5.4-5.6	4.61, 4.63, 4.75, 4.87, 4.91a, 4.92a
Introducing Entropy Entropy Chge in Rev. Proc.	6.1, 6.2, 6.3, 6.4	2.76, 2.85, 2.88, 5.34, 5.38, 5.48
Entropy Bal. in Closed Sys.	6.5	6.1, 6.2, 6.7
Entropy Bal. for Ctrl. Vol. Isentropic Processes	6.6, 6.7	6.23, 6.28a-c, 6.30a-c
Isentropic Efficiencies	6.8	6.63, 6.76, 6.83
Internally Rever. Proc. Ideal Gas Mixtures	6.9, 12.1-12.3	6.92, 6.95, 6.103, 6.132a,c,d, 6.140, 6.141a
Analysis of Mixture Sys.	12.4	6.142, 6.150, 6.153
Introducing Psychrometrics Psychrometric Chart	12.5 12.6, 12.7	6.160, 6.161, 6.169, 12.2,12.5,12.7
Introducing Combustion	13.1	12.11,12.14,12.26
Energy Bal., Reacting Sys.	13.2	12.43,12.47, 12.48, 12.65, 12.66
Reacting Mix. and Comb.		13.2,13.6,13.12
Vap. Po. Sys. – Rank. Cyc. Superheat and Reheat	8.1, 8.2, 8.3	13.39, 13.46, 13.50, 13.53
Regeneration Int. Comb. Eng., Otto Cycle	8.4, 9.1, 9.2	8.6, 8.14
Diesel and Dual Cycles	9.3, 9.4	8.2, 8.17, 8.28
Bray., Stirl., & Eric. Cyc. Vapor Comp. Refrig. Sys.	9.5, 9.6, 9.11, 10.1, 10.2	8.35, 8.45, 9.2, 9.25abcd
Actual Refrig. Sys. & Prop., Heat Pump Systems	10.3	9.32, 9.49, 9.93
	10.6	10.9,10.16abc,10.19 10.21, 10.23, 10.33, 10.35

The Energy Equation, Boundary Layer, Differential Operator and Navier-Stokes Equation were obtained from the Munson's textbook.

IV. Energy Equation

By applying Reynolds transport theorem to 1st Law of Thermo

$$\left(\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}\right) \text{ for system}$$

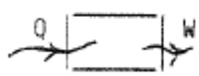
Let B = energy E

$$\beta = \frac{dE}{dm} = e = \underset{\text{internal energy}}{u} + \underset{KE}{\tfrac{1}{2}V^2} + \underset{PE}{gz}$$

Reynolds transport eq.:

$$\frac{d}{dt}(B_{sys}) = \frac{d}{dt}\left(\int_{cv} \beta\rho d\, Vol\right) + \int_{cs} \beta\rho(\rho\vec{v}\cdot\vec{n})\, dA$$

=> 1st Law for cv



$$\underset{\text{heat added to system}}{\frac{dQ}{dt}} - \underset{\text{WD by system}}{\frac{dW}{dt}} = \frac{dE}{dt} = \frac{d}{dt}\left[\int_{cv} e\rho d\,(Vol)\right] + \int_{cs} e\underbrace{\rho(\vec{v}\cdot\vec{n})\, dA}_{\dot{m}}$$

If assume:
1. no heat transfer
2. steady
3. 1-D inlet and outlet ($\dot{m} = \dot{m}_{out} = \dot{m}_{in}$)
4. incompressible

$$-\dot{w} = \dot{m}e_{out} - \dot{m}e_{in}$$

divided by $\dot{m}$ $\quad -w = e_2 - e_1 = (u + \tfrac{1}{2}v^2 + gz)_2 - (u + \tfrac{1}{2}v^2 + gz)_1$

Consider w, 2 components:

a. Shaft work (w_s) - work done by a machine (pump, compressor, piston, etc.) protruding thru cs to cv

b. Pressure work (w_p) - due to pressure forces at cs, can be accounted for by replacing internal energy (u) with enthalpy (h) on RHS of eq.

i.e. $h = u + P/\rho$

Steady-flow energy equation (energy per unit mass)

$$-w = (u_2 + \frac{1}{2}v_2^2 + gz_2) - (u_1 + \frac{1}{2}v_1^2 + gz_1)$$

$$-w_s = (h_2 + \frac{1}{2}v_2^2 + gz_2) - (h_1 + \frac{1}{2}v_1^2 + gz_1)$$

$$-w_s = \left(\frac{p_2}{\rho} + u_2 + \frac{1}{2}v_2^2 + gz_2\right) - \left(\frac{P_1}{\rho} + u_1 + \frac{1}{2}v_1^2 + gz_1\right)$$

divided by g

$$-\frac{w_s}{g} = \left[\frac{p_2}{\rho g} + \frac{u_2}{g} + \frac{v_2^2}{2g} + z_2\right] - \left[\frac{p_1}{\rho g} + \frac{u_1}{g} + \frac{v_1^2}{2g} + z_1\right]$$

(unit: length or head)

or

$$\frac{p_1}{\rho g} + \frac{u_1}{g} + \frac{v_2^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{u_2}{g} + \frac{v_2^2}{2g} + z_2\right] + \frac{w_s}{g}$$

where w_s/g is a measure of energy/(mass of fluid) transferred to the fluid due to shaft work (negative for a pump, positive for a turbine). We will refer to this as the head change due to shaft work, h_s (given in ft or in).

The equation can be rearranged

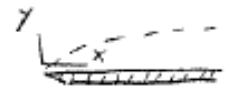
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2\right] + \underbrace{\frac{w_s}{g}}_{h_s} + \frac{u_2 - u_1}{g}$$

for steady, incompressible flow with friction, $(u_2 - u_1) > 0$, is a measure of loss of available energy due to friction losses. We will define a fricitonal head loss $h_f \equiv (u_2 - u_1)/g$, so that the energy equation becomes

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2\right] + h_s + h_f}$$

Note: h_s - for pump, + for turbine.

II. **Boundary-layer flows**



A. Laminar flat-plate boundary layer

Exact solution for equation of motion + continuity (Blasius sol'n) neglect g, 2D, steady flow, zero pressure gradient ($dp/dx = 0$), incompressible

Continuity: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{———} (1)$$

Momentum (Navier-Stokes eq.)

$$x: \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right)$$

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad \text{———} (2)$$

BC at $y = 0$, $u = 0$

at $y = \infty$, $u = U$

•
•
•

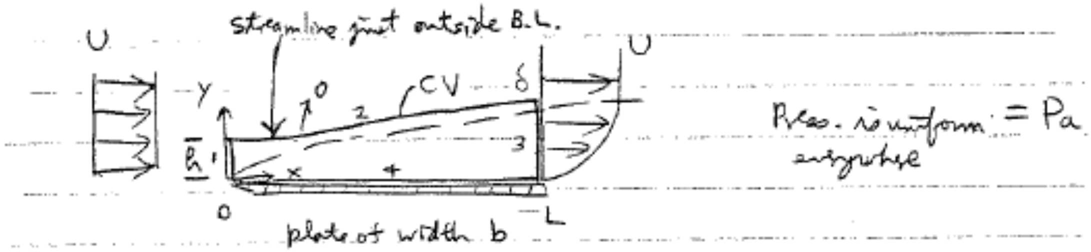
$$\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$$

wall shear stress coe, $c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$

II. **Boundary-layer flows - continued**

B. Momentum-integral estimates for BL

same assumptions as before, except for both laminar & turb. BL, instead of exact solution, use cv analysis



Momentum Eq. in x-dir (nonuniform vel. profile)

$$\Sigma F_x = \frac{d}{dt}(\;)^0 + \int u_x \, \rho(\vec{v}\cdot\vec{n}) \, dA$$

$$-F_D = \rho \int_1 u_x(\vec{v}\cdot\vec{n}) \, dA + \rho \int_3 u_x(\vec{v}\cdot\vec{n}) \, dA + \int_2{}^0 + \int_4{}^0$$

$$= \rho \int_0^h U\,(-U)\, b dy + \rho \int_0^\delta u(+u)\, b dy$$

$$F_D = \rho U^2 bh - \rho b \int_0^\delta u^2 \, dy$$

$h = ?$

Continuity (nonuniform vel. profile)

$$\int_{cv} \frac{d\rho}{dt}{}^0 \, d\,Vol + \int_{cs} \rho\,(\vec{v}\cdot\vec{n})\, dA = 0$$

$$\rho \int_0^h (-U)\, b dy + \rho \int_0^\delta u b dy = 0$$

$$\Rightarrow Uh = \int_0^{\delta} u\,dy$$

$$\therefore\ F_D = \rho b U \int_0^{\delta} u\,dy - \rho b \int_0^{\delta} u^2 dy$$

$$F_D = \rho b \int_0^{\delta} u(U-u)\ dy$$

$$F_D(x) = \rho b \int_0^{\delta(x)} u(U-u)\ dy$$

Note $\rho U^2 \theta \equiv \int_0^{\delta} \rho u(U-u)\ dy$ where θ = momentum thickness

$$\therefore\ F_D(x) = \rho b U^2 \theta \Rightarrow \frac{dF_D}{dx} = \rho b U^2 \frac{d\theta}{dx} \qquad \text{———— (1)}$$

However $F_D = b \int_0^{x} \tau_w(x)\ dx$ or $\frac{dF_D}{dx} = b\tau_w$ ———— (2)

Equating (1) and (2)

$$\Rightarrow \tau_w = \rho U^2 \frac{d\theta}{dx} \qquad \text{————(3)}$$

momentum-integral relation for both laminar or turbulent flow

For laminar flow, assume $u(y) = U(\frac{2y}{\delta} - \frac{y^2}{\delta^2})$,then evaluate $\theta = fn(\delta)$

$$\tau_w \equiv \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = fn(\delta)$$

⋮

sub θ, τ_w into (3) => diff. eq. => solving

Integral analysis	vs	Exact solution
$\frac{\delta}{x} \approx \frac{5.5}{\sqrt{Re_x}}$	compare to	$\frac{5.0}{\sqrt{Re_x}}$
$c_f \approx \frac{0.73}{\sqrt{Re_x}}$		$\frac{0.664}{\sqrt{Re_x}}$

SOME CONVENIENT DIFFERENTIAL OPERATORS IN FLUID MECHANICS

I. Total acceleration (substantial)

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \left[u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \right]$$

local acc. convective acc.

vector-gradient operator $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

so that $\vec{V}\cdot\nabla = (u\hat{i} + v\hat{j} + w\hat{k})\cdot\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

and $(\vec{V}\cdot\nabla)\vec{V} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$ (convective acc.)

$$\boxed{\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}}$$

local convective

II. Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\nabla\cdot(\rho\vec{V}) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left(\rho u\hat{i} + \rho v\hat{j} + \rho w\hat{k}\right) = \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0}$$

III. Momentum Equations

a. Euler's eq. (inviscid flow)

$$x:\ \rho g_x - \frac{\partial P}{\partial x} = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$

$$y:\ \rho g_y - \frac{\partial P}{\partial y} = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right]$$

$$z:\ \rho g_z - \frac{\partial P}{\partial z} = \rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right]$$

$$\nabla P = \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

$$\vec{g} = \hat{i}g_x + \hat{j}g_y + \hat{k}g_z$$

$$\boxed{\rho\vec{g} - \nabla P = \rho\frac{D\vec{V}}{Dt}}$$

b. Navier-Stokes eq.

$$x:\ \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$

$$y:\ \rho g_y - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right]$$

$$z:\ \rho g_z - \frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] = \rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right]$$

$$\rho\vec{g} - \nabla P + \quad ? \quad = \rho\frac{D\vec{V}}{Dt}$$

$$\nabla^2 = \nabla\cdot\nabla = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ (Laplacian operator)}$$

$$\Rightarrow \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\Rightarrow \quad ? \quad = \mu\nabla^2\vec{V}$$

=>

$$\boxed{\rho\vec{g} - \nabla P + \mu\nabla^2\vec{V} = \rho\frac{D\vec{V}}{Dt}}$$