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## Mathematics - 2

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### Problem 2: [100 Points]

#### Notes:

- Use of a calculator is not allowed
- You may optionally obtain one hint from the proctor to solve Part (a), if needed, for a 10-point penalty in your score.

A function  $y(x)$  is governed by the following differential equation and associated conditions.

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 4y = 0$$
$$y(1) = 2; y(x \rightarrow 0) \rightarrow \frac{1}{x^2}$$

- a. [40 Points] Solve for  $y(x)$ . Clearly show your solution steps.
- b. [20 Points] Determine the maxima and minima of  $y(x)$ .
- c. [10 Points] Determine the range of  $y$  that is **not** part of the solution.
- d. [10 Points] Determine the asymptote(s) of  $y(x)$ .
- e. [20 Points] On an  $x$ - $y$  plot, show the variation of the function  $y(x)$  with  $x$ , clearly identifying the information from Parts b and d above. Be neat in your drawing.