Problem 2: [100 Points]

Notes:
• Use of a calculator is not allowed
• You may optionally obtain one hint from the proctor to solve Part (a), if needed, for a 10-point penalty in your score.

A function \( y(x) \) is governed by the following differential equation and associated conditions.

\[
x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 4y = 0
\]
\[
y(1) = 2; \ y(x \to 0) \to \frac{1}{x^2}
\]

a. [40 Points] Solve for \( y(x) \). Clearly show your solution steps.
b. [20 Points] Determine the maxima and minima of \( y(x) \).
c. [10 Points] Determine the range of \( y \) that is not part of the solution.
d. [10 Points] Determine the asymptote(s) of \( y(x) \).
e. [20 Points] On an \( x \)-\( y \) plot, show the variation of the function \( y(x) \) with \( x \), clearly identifying the information from Parts b and d above. Be neat in your drawing.