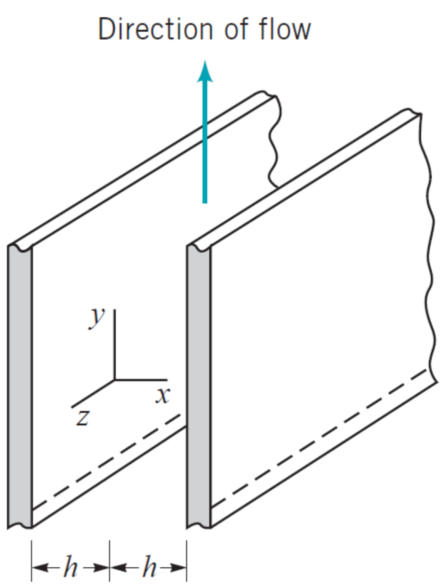
Fluid Mechanics - 2

A viscous, incompressible fluid flows between the two infinite, vertical, parallel plates with a gap of $2h$ as shown in the following figure. Assume that the flow is laminar, fully-developed, steady, uniform and no slip.

a. (**40 points**) Determine, by use of the conservation of momentum equations, an expression for the pressure gradient in the direction of flow.
b. (**40 points**) Determine the velocity in the gap of the parallel plates as a function of pressure gradient.
c. (**20 points**) Express pressure gradient in the direction of flow (from part a) in terms of the mean velocity in the gap.

Show all analysis and work, and mark answers clearly. (100 points in total)



Hint: In Cartesian coordinate system of (x, y, z), fluid velocity $\vec{V} = (u, v, w)$ and continuity equation for incompressible fluid is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of momentum equations for incompressible flow in Cartesian coordinate system with gravity $\vec{g} = (g_x, g_y, g_z)$, p is pressure, ρ is fluid density and μ is fluid viscosity:

$$-\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$-\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$-\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$