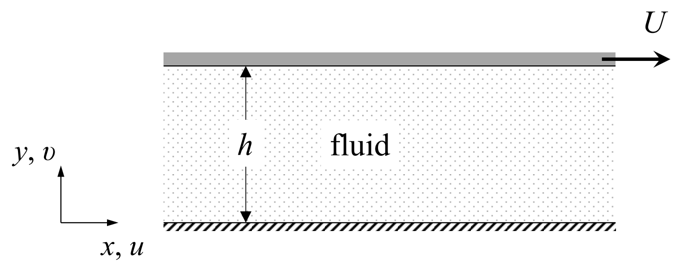
Consider a pressure-driven flow through a slot where the upper surface moves with a constant velocity U and the lower surface remains stationary. The top and bottom surfaces are infinite parallel plates in the x and z–direction and can be represented as a two-dimensional flow. The steady-state laminar parallel flow field can be found through the superposition of two solutions: Poisuielle flow (pressure-driven) and Couette flow (shear-driven). The steady-state velocity profile is:

$$u(y) = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - hy\right) + U\frac{y}{h}$$

where h is the distance of the slot height, dP/dx is the pressure gradient and μ is the constant fluid dynamic (absolute) viscosity.



[6 points] (a) Prove whether the flow is compressible or incompressible.

[5 points] (b) Find the general expression for the fluid shear stress $\tau(y)$ in terms of known variables U, h, dP/dx and μ.

[11 points] (c) For the case where $dP/dx = 0$, the solution reduces to Couette flow. Determine the position of the maximum velocity U_{max} and sketch the fluid shear stress for $0 \le y \le h$.

[11 points] (d) For the case where $U = 0$ and $dP/dx < 0$, the solution reduces to Poiseuille flow such that $dP/dx = -8\mu U_{max}/h^2$. Determine the position of the maximum velocity U_{max} and sketch the fluid shear stress for $0 \le y \le h$.

[12 points] (e) Determine the relationship between the maximum velocity U_{max} and mean velocity u_m. Begin with the definition of volume flow rate and find the relationship for Couette flow (part c) and Poiseuille (part d).

[5 points] (f) Based on the results in parts (c) and (d), is the fluid Newtonian or non-Newtonian? Briefly explain.

[50 points total]