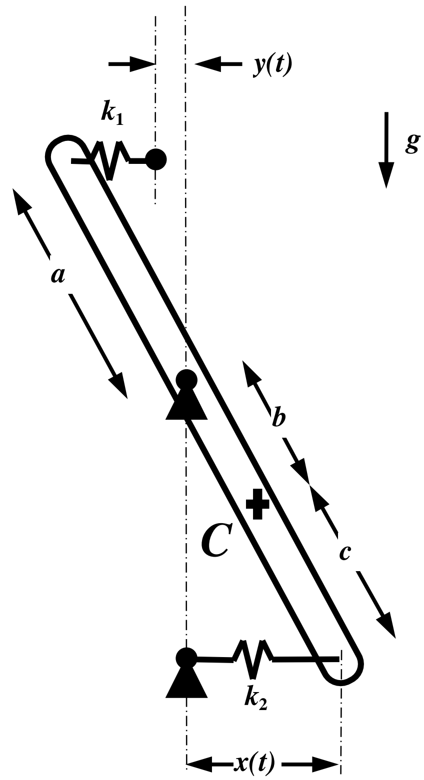

The system shown is in the vertical plane, so gravity is relevant. The rod is pinned at point O. The center of mass of the rod is at point C. The displacement $y(t)$ is a specified input. Use $x(t)$ as the output that defines the motion. The positive directions for x and y are as indicated by the arrows. The rod has moment of inertia J_O with respect to the pin joint and mass m. All friction is negligible. The rod is vertical when there are no forces in the stiffness elements. Assume small angle motions.

1. (35 points) Draw a complete free-body diagram. Derive a single equation of motion for $x(t)$ using the forces shown on your free-body diagram (that is, use Newton's laws and do not use Lagrange's equations or any other energy method). Define a natural frequency ω_n and parameter p such that you can write the equation as
$$\ddot{x} + \omega_n^2 x = p^2 y(t) \quad p^2 = \frac{k_1 a(b+c)}{J_O}$$

2. (40 points) Introduce a damping ratio ζ such that the model takes the form
$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = p^2 y(t)$$
Using this given notation, show all mathematical steps (showing the steps is as important as getting a correct final answer) to derive the response of the system for input of $y(t) = Y \sin \omega t$. Use of complex algebra is encouraged, because it is the simplest algebraically, but not mandatory. The final answer should be expressed in terms of a non-dimensional magnification function $M(\omega)$ and a phase angle $\phi(\omega)$ such that
$$x(t) = YM(\omega)\sin(\omega t - \phi).$$

3. (25 points) An engineer performs experiments on a system to obtain the attached figures. She then applies excitation $y(t)$ with amplitude of 0.5 mm. She measures that the response $x(t)$ lags the input displacement $y(t)$ by 160 deg. Write an expression for the response $x(t)$. What is the amplitude of the response $x(t)$ (give units)? Make sure your thought process is clear by using words, equations, and markings on the figures as necessary (that is, do not simply give an expression and a numerical value).