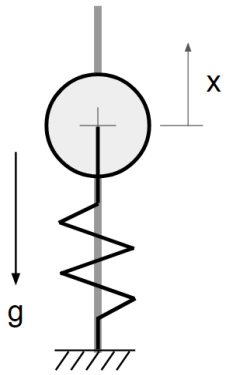
Consider a mass constrained to move along a frictionless pole parallel to gravity.




Degree of Freedom

- $x$: height of the center of the sphere measured from the point at which the spring is unstretched. That is
  - x=0 ⇔ spring is unstretched
  - The length of the spring at any moment is L + x

System Parameters

- $m_1$: mass of the sphere
- $m_s$: mass of the spring
- k: the stiffness of the spring
- r: radius of the sphere
- L: unstretched length of spring
- g: gravitational constant

Part 1 of 2 (70 points) - ***Assume the spring is massless***

A. (15 points) Derive expressions for all the potential and kinetic energy in the system in terms of the degree of freedom and its derivative and the system parameters.

B. (20 points) Use an energy method to derive the equation of motion.

C. (25 points) Assume the following form for the homogeneous solution

$$x_H = Acos(\omega_n t) + Bsin(\omega_n t)$$

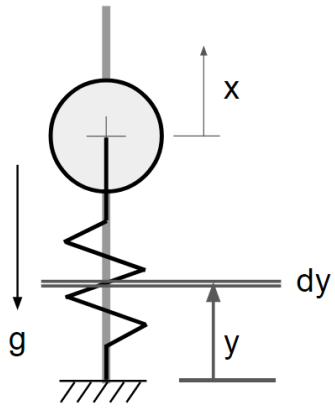
And derive the total solution (homogeneous plus particular) of $x$ for initial conditions

$$x(t=0) = x_0, \text{ and } \dot{x}(t=0) = v_0$$

D. (10 points) Derive the maximum deflection of the spring for initial conditions

$$x(t=0) = 0, \text{ and } \dot{x}(t=0) = 0$$

Part 2 of 2 (30 points)- ***Include the mass of the spring***




Assume the spring deflects linearly along its length and the total mass of the spring, $m_s$, is uniformly distributed along its length.

Specifically, consider an infinitesimal element, dy, at a location y above the ground. The velocity of this infinitesimal element at location y is

$$\dot{y} = \dot{x}\, y / (L + x)$$

The mass of this infinitesimal element is

$$dm = m_s\, dy / (L + x)$$

E. (10 points) Derive expressions for the potential and kinetic energy in the spring (not just the infinitesimal element) in terms of $x,\ \dot{x}$, and the system parameters. The final answer is not a function of y.

F. (10 points) Use an energy method to derive the equation of motion.

G. (10 points) Derive the static deflection.