



Consider a tank with an inflow pipe and an outflow pipe connected to it. The tank holds a mixture of liquids A and B. Within the tank, at time t there is a volume $V_A(t)$ liters of liquid A and a volume $V_B(t)$ liters of liquid B.

Initially, at time $t=0$ seconds, the tank is filled to a volume V_0 liters entirely with fluid B, i.e.

$$V_B(t=0) = V_0 \text{ and } V_A(t=0) = 0.$$

Starting at time $t=0$, fluid A flows into the tank through the inflow pipe at a rate of X liters/second, and the mixture of fluids A and B in the tank flows out of the outflow pipe at a rate of X liters/second. Since the inflow rate is the same as the outflow rate, the total volume inside the tank remains constant at V_0 , i.e.

$$V_A(t) + V_B(t) = V_0.$$

For this entire problem, assume that when fluid A flows into the tank, it is instantly dispersed evenly within the tank, so that the volume fractions of A and B in the outflow are the same as those in the tank.

- 1) (40%) Write differential equations for $V_A(t)$ and $V_B(t)$ in terms of V_0, X, t , and any mathematical functions. Be sure to show your work for how you figured out these equations.
- 2) (40%) Solve these differential equations for $V_A(t)$ and $V_B(t)$ in terms of V_0, X, t , and any mathematical functions.
- 3) (10%) Graph $V_A(t)$ and $V_B(t)$. Label all important points and asymptotes (if any).
- 4) (10%) Find the time when $V_A(t) = V_B(t)$. Make sure you express this as a positive number, i.e. there is no negative sign in front.