Consider the fuel cell stack problem. The $t = 0.21$ mm thick membranes have a nominal thermal conductivity of $k = 0.79\ W/m \cdot K$ that can be increased to $k_{eff} = 15.1\ w/m \cdot K$ by loading 10%, by volume, carbon nanotubes into the catalyst layers. The membrane experiences uniform volumetric energy generation at a rate of $\dot{q} = 10{\times}10^6\ W/m^3$. Air at $T_a = 80\ °C$ provides a convection coefficient of $h_a = 35\ W/m^2 \cdot K$ on both sides of the membrane. The flow channels are $2L = 3$ mm long. The membrane is clamped between plates, each of which is at a temperature $T_p = 80\ °C$.

a. (40) Derive the differential equation that governs the temperature distribution T(x) in the membrane. Ignore the temperature difference in the y direction. Assume there are no variations in the normal (z) direction.

b. (30) Obtain a solution to the differential equation, assuming the membrane is at the plate temperature at $x = 0$ and $2L$. At x = L, the temperature gradient is 0.

c. (30) Use the solution from part b to calculate the maximum temperature at T(L) and sketch the temperature distribution T(x) from $x = 0$ to $x = L$ for carbon nanotube loadings of 0% and 10% by volume. Comment on the ability of the carbon nanotubes to keep the membrane below its softening temperature of 85°C.