PhD Qualifying Exam
Nuclear Engineering Program

Part 1 – Core Courses
(Solve 3 problems only)

9:00 am – 12:00 noon, October 27, 2017
(1) Nuclear Reactor Analysis

The Multigroup diffusion equations for a multiplying medium is given by

\[-D_g \nabla^2 \phi_g + \Sigma_{R,g} \phi_g = \frac{X_g}{k} \sum_{g'=1}^{G} \nu \Sigma_{f,g'} \phi_{g'} + \sum_{g'=1}^{G} \Sigma_{s,g' \rightarrow g} \phi_{g'} \]

a. (37%) Derive the 3-group form of the above equations, considering the following conditions: Direct coupling, no up-scattering, fission neutrons are generated in the first group, and only third-group neutrons cause fission.

b. (29%) Derive the criticality condition (i.e., a formulation for \(k\)) for a bare reactor.

c. (34%) Identify different terms in the \(k\) formulation similar to the parameters in the six-factor formula.
Consider a typical PWR fuel rod,

a. (20%) Write the general heat conduction equation.

b. (20%) Obtain a simplified equation by assuming:
   - constant thermal conductivity and specific heat
   - steady state with a uniform volumetric power generation rate $q''$
   - no axial conduction
   - axisymmetry and using
   \[
   \nabla^2 T = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}
   \]

c. (40%) If the bulk coolant temperature is given as $T_m$, explain briefly the solution procedure to obtain the maximum temperature in the fuel (need not to solve, oxide layer on the cladding surface can be neglected).

d. (20%) Sketch the radial temperature profile for the entire fuel rod.
(3) Advanced Nuclear Materials

a. (30%) What is the Nil-ductile transition temperature?
b. (40%) How does the Nil-ductile transition temperature change under neutron irradiation and why?
c. (30%) Why the Nil-ductile transition temperature is a very important mechanical property to study for the reactor vessel?
(4) Radiation Detection and Shielding

Detector Dead Time

Two models of dead time behavior of counting systems have come into common usage: paralyzable and nonparalyzable response. We will use the following notation:

\[ n = \text{the true interaction rate (or count rate) of radiation in the detector,} \]
\[ m = \text{the recorded or observed count rate by the detector, and} \]
\[ \tau = \text{the system dead time following a radiation detection event.} \]

Note that due to the dead time of the detector some of the true interactions are not counted so typically \( m < n \).

For the nonparalyzable model, if we know the dead time \( \tau \) we can calculate the true rate \( n \) from the measured rate \( m \) as follows: \( n - m = nm\tau \), or solving for the true count rate \( n = \frac{m}{1 - m\tau} \).

For the paralyzable model, we can calculate the true rate \( n \) from the measured rate \( m \) through a process of iteration to solve for \( n \) in the following equation: \( m = ne^{-n\tau} \).

(a) (70%) There are two methods for measuring the dead time of a detector. One is the split source method and another is the decaying source method. We will use the decaying source method in this problem. For the decaying source method we use a radioisotope that has a fairly short half-life and take multiple measurements of the source as it decays resulting in fewer measured counts \( m \) on each subsequent count. If we assume the background counts are negligible, we then know that \( n_1 = n_0 e^{-\lambda t} \), where \( n_0 \) is the true count rate at the beginning of the measurement, \( \lambda \) is the decay constant of the particular radioisotope used in the measurement, and \( n_1 \) is the true count rate after time \( t \) has elapsed.

A radioisotope source of \(^{116m}\text{In}\) with a half-life of 54.3 min is used for our dead time measurement. Successive observations with a counting period of 1 minute result in 131,340 counts at 12:00 noon and 93,384 counts at 12:40. Neglecting background and assuming a nonparalyzable model for dead time losses, calculate the true interaction rate \( n_0 \) in the detector at 12:00 noon.

(b) (15%) What is the dead time for this detector in milliseconds?

See next page for part (c)
(c) (15%) A plot of the observed rate $m$ versus the true rate $n$ is given in Figure 1 below for both the nonparalyzable and paralyzable models. Mistakes in the interpretation of nuclear counting data from paralyzable systems (such as Geiger-Müller counters) have occurred in the past by overlooking the fact that there are always two possible true interaction rates corresponding to a given observation rate. As shown in Figure 1, the observed rate $m_1$ can correspond to either true rates $n_1$ or $n_2$. In practice, how would you resolve this ambiguity to determine which is the correct true rate?

Figure 1
The probability density function for speed of particles at temperature (T) is expressed by

\[ f(v)dv = \alpha v^2 e^{-\frac{mv^2}{2kT}} dv \]

where, \( \alpha \) is a constant coefficient.

a) (25%) Derive a formulation for the most probable speed of particles.

b) (25%) Derive a formulation for the density function in terms of kinetic energy (E).

c) (20%) Does the most probable speed correspond to the most probable energy? Explain your answer.

d) (30%) Derive a formulation for the average energy of particles.
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Part 2 – Specialty Part
Nuclear Fuel Cycle and Radioactive Waste Management

Answer 3 questions only by answering both Questions 1 & 2
and then answering either Question 3 or Question 4

2:00 pm – 5:00 pm, October 27, 2017
Specialty - Nuclear Fuel Cycle and Radioactive Waste Management

The student must complete both Questions #1 and #2 below.

Question #1 (30%)  
Draw a simple diagram showing a closed nuclear fuel cycle when a sodium-cooled fast reactor (Actinide metal fuel, for example U-Pu-Zr-Ac) is used from the beginning of the front-end to the end of back end. Explain the functions of each step.  
For enriching U, which U-chemical compound is selected as the major species for processing and explain the reason why this compound is selected and the three technologies for enrichment including their mechanisms.

Question #2 (30%)  
The PUREX process was the source of most of the high-level radioactive waste generated in the U.S.  
(a) Outline and describe the major steps in the PUREX process. Include in your description the primary reagents used at each step.  
(b) For each major step in the PUREX process, discuss what radioactive waste types are produced.

The student must select one of the following two questions.

Question #3 (40%)  
For an electrorefiner cell, the anode is pure metal Pu and the cathode is an inert electrode. The salt is KCl-LiCl-UCI3 with UC13 mole fraction X UC13. The applied current is I. During operation, the surface area of the anode (Aa) and the cathode (Ac) are kept constant, and the anodic and cathodic reactions are at equilibrium.  
Give the expression of the cell potential at the time when the current is applied (the difference between cathode and anode)  
Give the expression of operation time at which Pu starts depositing at the Cathode.  

The following constant parameters are known:  
The salt density ($\rho_m$), salt mole weight ($M_m$), total salt volume ($V_m$), the diffusion coefficient of Pu ($D_{Pu}$), the diffusion coefficient of U ($D_U$), the diffusion layer at the anode ($\delta_a$), the diffusion layer at the Cathode ($\delta_C$), the standard potential U$^{3+}$/U ($E_{U^{3+}}^0$), the standard potential Pu$^{3+}$/Pu ($E_{Pu^{3+}}^0$), the activity coefficient of Pu$^{3+}$ ($\gamma_{Pu^{3+}}$) and U$^{3+}$ ($\gamma_{U^{3+}}$).  

All the parameters are in SI units.  

See next page for Question #4
**Question #4 (40%)**

For single-layer oxide layer on steel with a constant scale removal rate $R$, a parabolic constant $K_p$ and an initial thickness 0, the oxide layer has a thickness $X$ after time $t$, give the weight change after time $t$ of steel per unit area (note: $\rho_{ox}$ is the oxide density and $f_o$ is the weight fraction of oxygen in the oxide) and deduce the expression for the maximal weight change.

Based on the expression you obtain, deduce the weight change rate. When $t \to \infty$, show the expression that the rate approaches.

All the parameters are in SI units.
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Part 2 – Specialty Part
Thermal Hydraulics and Reactor Safety

2:00 pm – 5:00 pm, October 27, 2017
Specialty – Thermal-hydraulics and Reactor Safety (1) (30%)

a. Sketch the Nukiyama pool boiling curve. [30%]
b. Identify the following characteristic points: onset of nucleate boiling (ONB), critical heat flux (CHF), and Leidenfrost point. [15%]
c. Explain briefly the different heat transfer modes in the boiling curve. [15%]
d. Explain the difference between power-controlled and temperature-controlled pool boiling experiments. [20%]
e. Describe the CHF mechanisms of Boiling Water Reactor (BWR) and Pressurized Water Reactor (PWR). [20%]
Consider a viscous liquid flowing down an inclined plate, the following conditions are given:

- The flow is laminar, steady-state, and fully-developed.
- The angle between the infinitely large plate and horizontal plane is $\theta$.
- Viscosity $\mu$ and density $\rho$ of the liquid are constants.
- No-slip condition can be assumed at the wall boundary.
- The gas has a constant pressure $p_\infty$.
- The gas-liquid interface is flat, and the shear at the interface is negligibly small.

**Specialty – Thermal-hydraulics and Reactor Safety (2) (35%)**

a. Write the continuity equation. [10%]
b. Write the momentum equation in the $x$-direction, specify relevant boundary conditions. [20%]
c. Write the momentum equation in the $z$-direction, specify relevant boundary conditions. [20%]
d. Explain briefly the solution procedure to obtain the $z$-component velocity $v_z$ (need not to solve), sketch the velocity profile. [30%]

Hint: $v_z$ can be given as $v_z = \frac{p g \delta^2 \sin \theta}{2 \mu} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right]$.

e. If the volumetric flow rate with a width $W$ (in the $y$ direction) is given as $Q$, obtain an expression of the film thickness $\delta$. [20%]
Specialty – Thermal-hydraulics and Reactor Safety (3) (35%)

Consider a pressurized water reactor (PWR) nuclear power plant,

a) Draw the line diagram and identify the major components of the primary and secondary loops. [30%]
b) List the major engineered safety features. [15%]
c) Explain the fission product barriers. [15%]

Write the control volume balances of mass and energy for the primary loop, explain loss of coolant accident (LOCA) using these equations. Describe the mass and energy inventory changes following