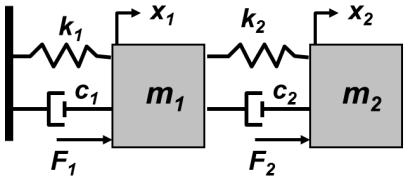
The mass and stiffness matrices of a mechanical system are given by

$$[M] = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} kg \qquad [K] = \begin{bmatrix} 30 & -5 \\ -5 & 10 \end{bmatrix} \frac{N}{m}$$



Part a (50 pts):

1. (20 pts) Compute natural frequencies in rad/sec and mode shapes.
2. (5 pts) Sketch the 2nd mode shape.
3. (10 pts) Compute modal mass and stiffness for first mode.
4. (10 pts) Demonstrate that the two modes are orthogonal with respect to mass matrix.
5. (5 pts) Are the modes orthogonal with respect the stiffness matrix:

   YES or NO

Part b (50 pts):

1. (10 pts) Please describe the motion of the displacements $x_1(t)$ and $x_2(t)$ to the initial conditions

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \qquad \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

NOTE: Assume the system has no damping. Thus, the equation of motion is:

$$[M]\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} + [K]\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

2. (10 pts) If you would like for the system to respond only in the first mode (e.g. no contribution of the 2nd mode to the response), what displacement initial conditions do you need to impose?

NOTE: Assume initial velocities are zero: $\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

3. (30 pts) Compute the steady-state harmonic responses $x_1(t)$ and $x_2(t)$

$$[M]\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} + [K]\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 10\cos(2t) \\ 0 \end{Bmatrix}$$