The mathematical model of a system is:

Equations of motion

$$m\ddot{x} + k_1 x + k_2(x - y) = p(t)$$

$$b_2\dot{y} + k_2(y - x) = 0$$

Initial Conditions

$$x\,(0) = \dot{x}(0) = 0$$

$$y(0) = 0$$

In general, the transfer function is defined as the Laplace transform of the output divided by the Laplace transform of the input

$$G(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}$$

The transfer function is defined for zero initial conditions.

a. (55 points) Determine the transfer functions
   $G_x(s) = \frac{X\,(s)}{P\,(s)}$ and $G_y(s) = \frac{Y\,(s)}{P\,(s)}$
   where $X(s) = \mathcal{L}[x(t)]$, $Y(s) = \mathcal{L}[y(t)]$, and $P(s) = \mathcal{L}[p(t)]$.
b. (10 points) What do the transfer functions in Part (a) reduce to for the special case where $b_2 = 0$?
c. (35 points) Determine the solutions $x(t)$ and $y(t)$ for the special case where $b_2 = 0$ and the applied force is a step function, $p(t) = H(t)$.

A table of Laplace Transform pairs and properties is attached.

Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}[F(s)]$	$F(s) = \mathcal{L}[f(t)] = \int_{t=0}^{\infty} f(t)e^{-st}\,dt$
Impulse, $\delta(t)$	1
Unit step, $H(t)$	$\frac{1}{s}$
Ramp, t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}cos(\omega t)$	$\frac{a+s}{(s+a)^2+\omega^2}$

Properties of Laplace Transforms

Proportionality or scaling	$\mathcal{L}[a \cdot f(t)] = a \cdot \mathcal{L}[f(t)] = a \cdot F(s)$
Superposition or addition	$\mathcal{L}[f_1(t) + f_2(t)] = \mathcal{L}[f_1(t)] + \mathcal{L}[f_2(t)]$ $= F_1(s) + F_2(s)$
First derivative	$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = \mathcal{L}\left[\dot{f}(t)\right] = sF(s) - f(0)$
Second derivative	$\mathcal{L}\left[\frac{d^2}{dt^2}f(t)\right] = \mathcal{L}\left[\ddot{f}(t)\right] = s^2F(s) - sf(0) - \dot{f}(0)$
Multiplication by e^{-at}	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
Translation	$\mathcal{L}[f(t-a)H(t-a)] = e^{-as}\mathcal{L}[f(t)] = e^{-as}F(s)$