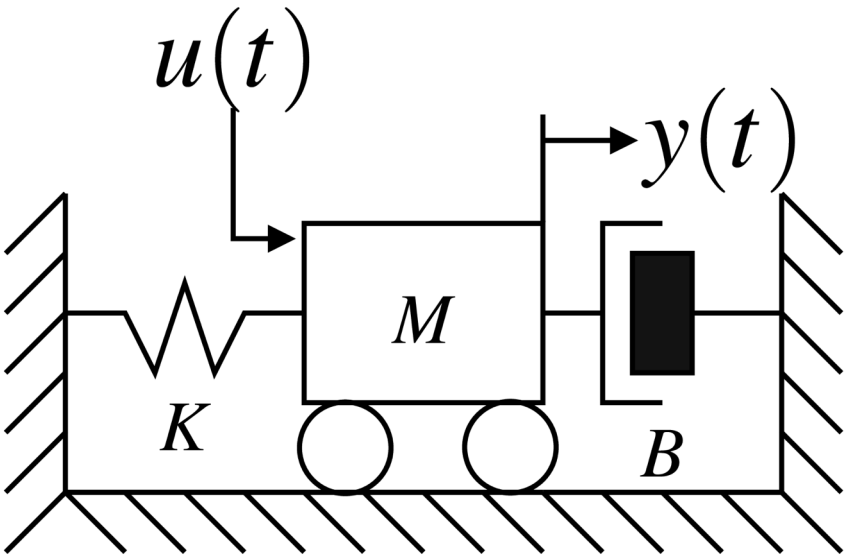

Illustration 1: Mechanical system (friction-free cart).

In the mechanical system shown in Figure 1, the system input is the force u(t) and the output the displacement y(t). The physical system parameters are stiffness K , mass M , and damping B.

1. Write down the differential equation for the system in terms of the physical system parameters (K , M , B). (10 points)

2. Write down the differential equation for the free vibration response of the system in terms of natural frequency (ω_n) and damping ratio (ζ) and express these quantities (ω_n, ζ) in terms of the physical system parameters. (10 points)

3. Qualitatively sketch the free vibration response of the system for $\omega_n = 4\pi$ and $\zeta = 1/8$ in the blank graph provided in Figure 2. Assume that the mass is released from an initial displacement of 0.2 meters and zero initial velocity. Add numbers to the grid lines on the x- and y-axis of the graph; the sketch of response function can be approximate. (20 points)

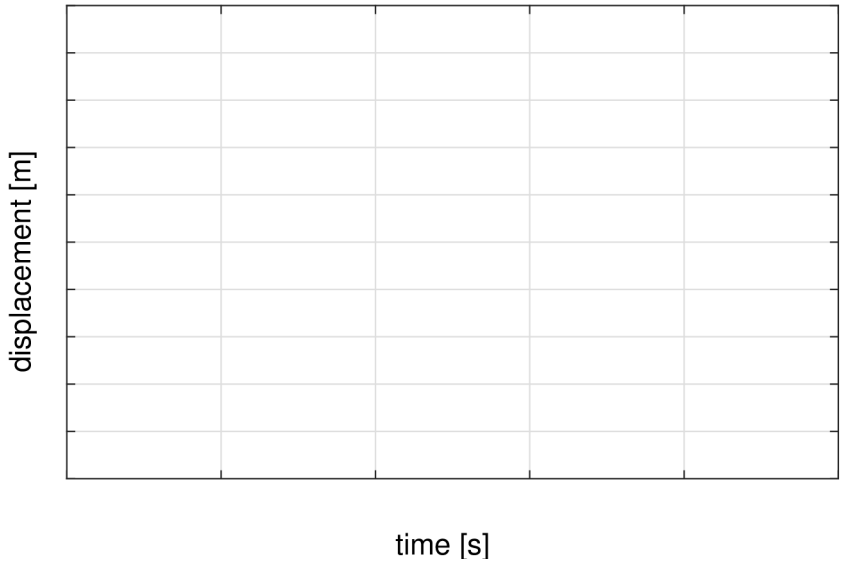

Illustration 2: Free vibration response of the system in Figure 1 for $\omega_n = 4\pi$ and $\zeta = 1/8$.

4. Derive the state space representation for the system. (25 points)

5. Using the force-current electro-mechanical analogy, replace the force u(t) with a current i(t) in the differential equation of the system. What are the resulting electrical analogs of displacement x(t), stiffness K , mass M , and damping B ? (15 points)

6. Sketch the circuit of the analog electrical system of the mechanical system in Figure 1 using the force-current electro-mechanical analogy. (20 points)