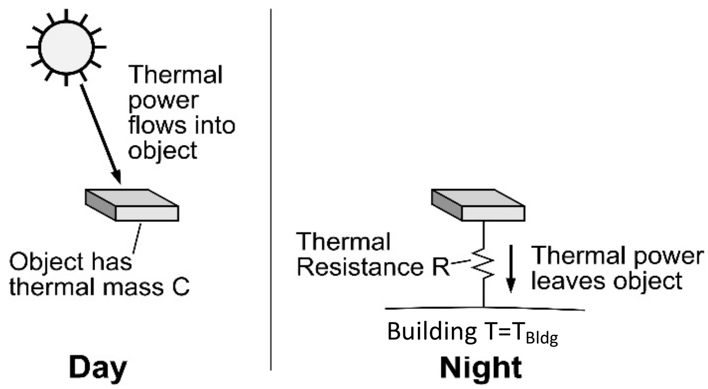
In one type of passive solar heating system, a mass is heated by the sun during the day and then is used to warm a building during the night.




The left diagram illustrates the system during the day. The power delivered during the day varies according to the angle of the sun:

$$P_{Day} = P_{Noon}\sin(2\pi t)$$

Here $P_{Noon}$ is the maximum power [Joules/day] and $t$ is the time [days] since sunrise.

The right diagram illustrates the system at night. The power removed (negative) from the system depends on the temperature difference between the mass and the building:

$$P_{Night} = -\frac{T_{Mass} - T_{Bldg}}{R}$$

Here $T_{Mass}$ is the temperature of the heated mass (varying with time) and $T_{Bldg}$ is the temperature inside the building (constant), both with units °C. $R$ is the thermal resistance between the mass and the building [days*°C/Joule].

Both day and night, the temperature change of the mass is given by

$$\frac{dT_{Mass}}{dt} = \frac{P}{C}$$

Here $P$ is $P_{Day}$ or $P_{Night}$, depending on the time, and $C$ is the thermal capacity of the mass [Joules/°C].

During the day the mass is isolated from the buiding, and during the night energy loss from the mass to the outdoors is negligible. The day and night are each half a day long, eg $P = P_{Day}$ for $0.0 < t < 0.5$ day and $P = P_{Night}$ for $0.5 < t < 1.0$ day.

a. (10 Points) Find (and sketch) the equation for the temperature of the mass during a single day (for $0.0 < t < 0.5$ day). Label the temperature at sunset on the graph. You can assume the temperature at sunrise is $T_{Day,0}$ .
b. (10 Points) Find (and sketch) the equation for the temperature of the mass during a single night (for $0.5 < t < 1.0$ day). Label the temperature at sunrise on the graph. You can assume the temperature at sunset is $T_{Night,0.5}$ .
c. (10 Points) When the system is at cyclic steady state, the temperature at the end of the day is the temperature at the beginning of the night, while the temperature at the end of the night is the temperature at the beginning of the day. Use these constraints to determine the temperature of the mass at sunrise $(T_{Day,0})$ as a function of $C$, $P_{Noon}$, $R$, and $T_{Bldg}$ when the system is at cyclic steady state.