

You are responsible for designing a new shaft with the loading conditions as shown in the figure above.

- The shaft supports an 8-kip bending force.
- Bearing reactions  $R_1$  and  $R_2$  are exerted on the shaft as shown in the figure.
- The shaft rotates at 950 rev/min.
- The shaft is made of 1095 HR steel.
- The shaft's surfaces are machined.

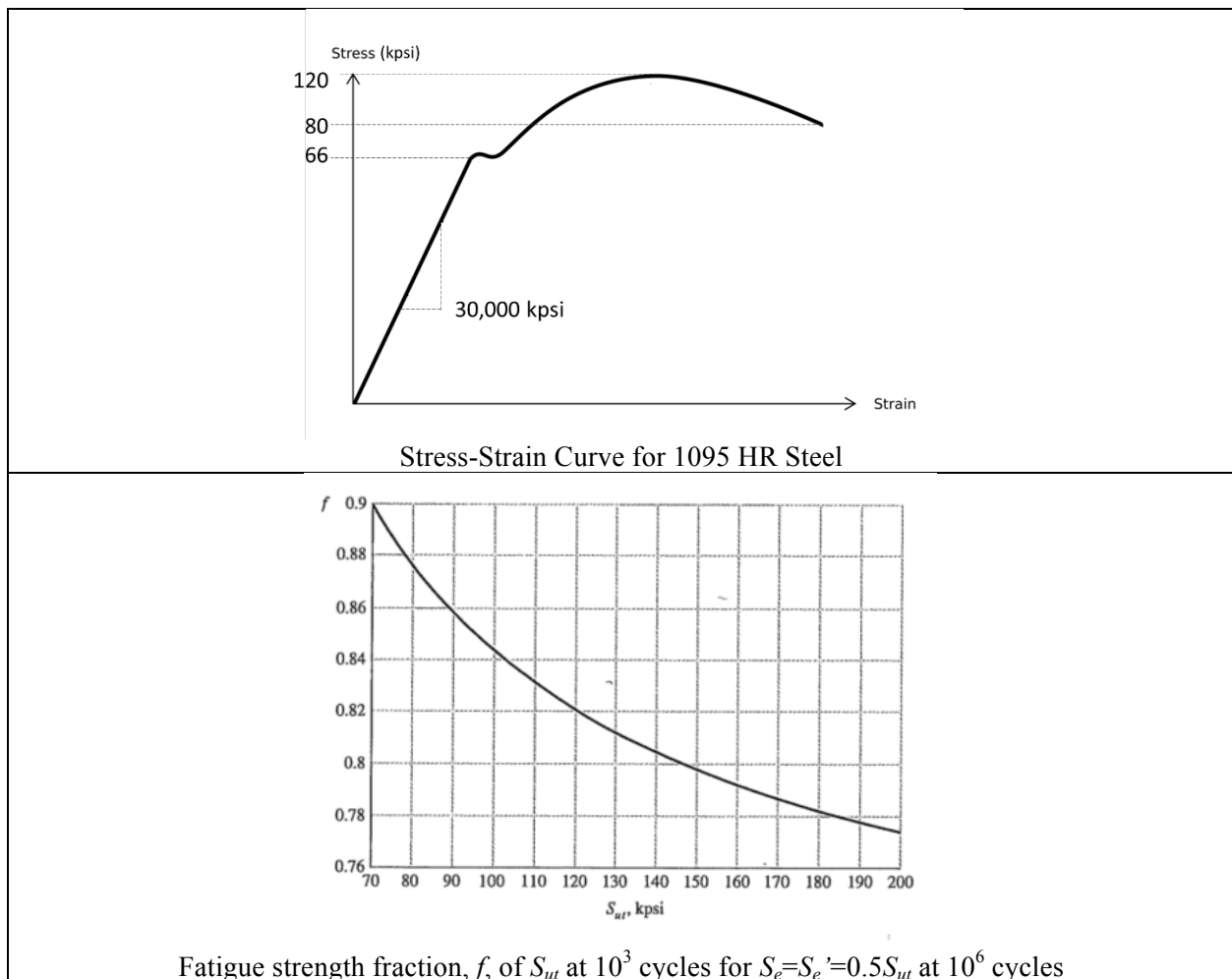
Please answer the following questions about the shaft design.

- a) (5 points) What is the ultimate tensile strength and yield strength of the 1095 HR steel? (see the stress-strain curve on the following page)
- b) (10 points) What are reaction forces,  $R_1$  and  $R_2$ ? Plot the shear force and bending moment diagrams.
- c) (20 points) Calculate effective maximum stress in fatigue. (Hint: you will need to identify the fatigue stress-concentration factor,  $k_f$ )
- d) (25 points) Calculate endurance limit,  $S_e$ . Be sure to identify and calculate the relevant endurance limit modifying factors (also known as Marin factors).
- e) (25 points) Determine the fatigue strength,  $S_f$ .
- f) (15 points) Determine the factor of safety of the design for a 10 hour operating life.

Potentially helpful equations:

<ul style="list-style-type: none"> <li>• <math>I_{cylinder} = \frac{\pi d^4}{64}</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>k_a = a S_{ut}^b</math></li> </ul>
<ul style="list-style-type: none"> <li>• <math>\sigma = \frac{Mc}{I}</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>k_c = 1</math> (bending) = 0.85 (axial) = 0.59 (torsion)</li> </ul>
<ul style="list-style-type: none"> <li>• <math>S'_e = 0.5 S_{ut}</math> (<math>S_{ut} &lt; 200</math> kpsi) = 100 kpsi (<math>S_{ut} &gt; 200</math> kpsi) = 700 MPa (<math>S_{ut} &gt; 1400</math> MPa)</li> </ul>	<ul style="list-style-type: none"> <li>• <math>k_d = \frac{S_T}{S_{RT}}</math></li> </ul>
<ul style="list-style-type: none"> <li>• <math>S_f = a N^b</math> <ul style="list-style-type: none"> <li>○ <math>a = (f S_{ut})^2 / S'_e</math></li> <li>○ <math>b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S'_e}\right)</math></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• <math>k_f = 1 + q(k_t - 1)</math></li> </ul>

Potentially helpful Figures and Tables:



**Table 6-2**

Parameters for Marin Surface Modification Factor, Eq. (6-19)

Surface Finish	Factor $a$		Exponent $b$
	$S_{UT}$ , kpsi	$S_{UT}$ , MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

For axial loading there is no size effect, so

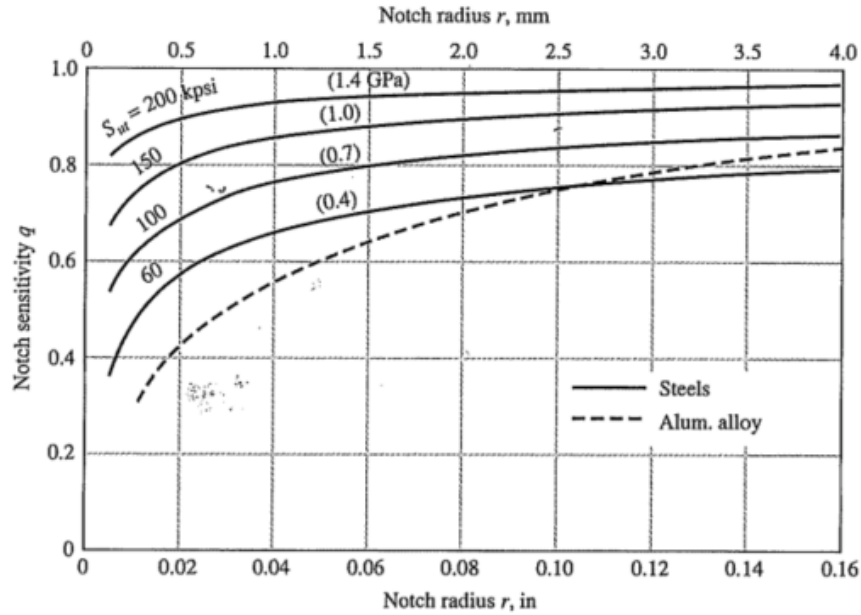
$$k_b = 1$$

Temperature, °C	$S_T/S_{RT}$	Temperature, °F	$S_T/S_{RT}$
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

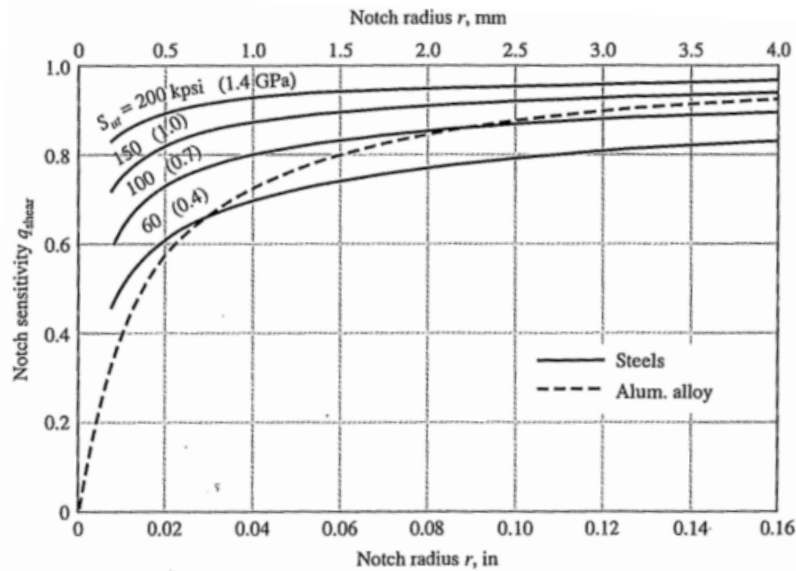
Effect of operating temperature on the tensile strength of steel ( $S_T$  = tensile strength at operating temperature;  $S_{RT}$  = tensile strength at room temperature)

Reliability, %	Transformation Variate $z_\alpha$	Reliability Factor $k_e$
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Reliability Factors  $k_e$  Corresponding to 8 percent standard deviation of the endurance limit



Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of  $q$  corresponding to the  $r = 0.16$ -in (4-mm) ordinate.



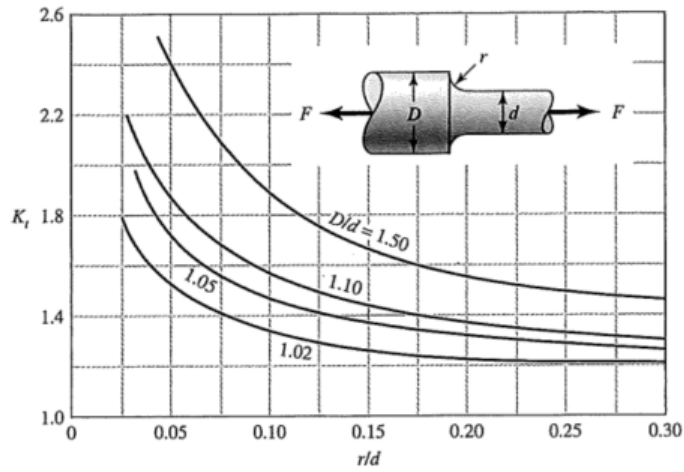
Notch-sensitivity charts for materials in reversed torsion. For larger notch radii, use the values of  $q_{shear}$  corresponding to  $r = 0.16$ -in (4-mm).

**Table A-15**

Charts of Theoretical Stress-Concentration Factors  $K_t^*$  (Continued)

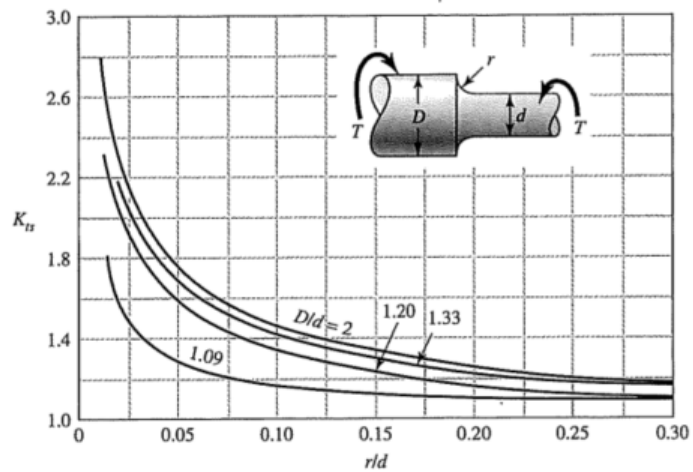
**Figure A-15-7**

Round shaft with shoulder fillet  
in tension.  $\sigma_0 = F/A$ , where  
 $A = \pi d^2/4$ .



**Figure A-15-8**

Round shaft with shoulder fillet  
in torsion.  $\tau_0 = Tc/J$ , where  
 $c = d/2$  and  $J = \pi d^4/32$ .



**Figure A-15-9**

Round shaft with shoulder fillet  
in bending.  $\sigma_0 = Mc/I$ , where  
 $c = d/2$  and  $I = \pi d^4/64$ .

