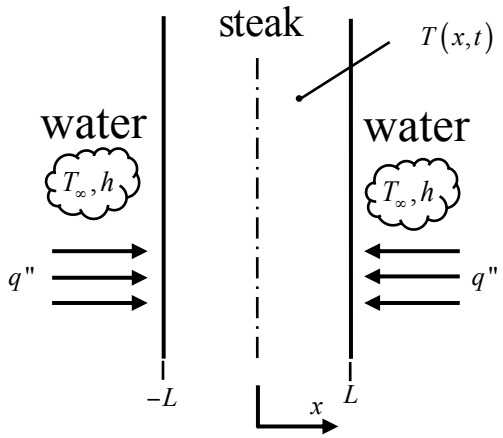
Heat Transfer -1

"Sous Vide" is a method of cooking in which food is cooked in a water bath at precisely controlled temperature. A person is going to sous vide a 3 cm thick slab of steak in a water bath maintained at a constant temperature of $T_\infty = 60$ °C. The initial temperature of the steak is $T_i = 4$ °C. The heat transfer coefficient on the surface of the steak is $h = 95$ W/m^2· K. The steak has constant thermophysical properties: density $\varrho = 1067$ kg /m^3, specific heat $c_p = 3810$ J/kg· K, and thermal conductivity $k = 0.49$ W/m· K. Assume that there are no chemical reactions and fluid loss from the meat is negligible.

1. Show the proper differential equation for the transient temperature as a function of position T(x,t) inside the steak during the cooking process. Identify the necessary initial and boundary conditions to solve the problem, but do not perform the solution. [25%]
2. If the steak remains in the water bath for a sufficiently long time, sketch the temperature profiles at 0 seconds, 10 minutes, 30 minutes and 72 hours after the steak is placed in the water bath. Clearly identify each of the four curves. [20%]
3. The cooking is considered complete when the minimum temperature inside the steak reaches 54 °C. Determine how long this process will take. [30%]
4. Sketch the variation of heat flux at the steak-water interface as a function of time. [10%]
5. Calculate heat flux from the water to the steak at times of 0 seconds and 30 minutes. [15%]



Note:

- The approximate solution of the transient temperature profile inside a $2L$ thick plane wall subject to convective heat transfer on both sides (see figure above) is given by:

$$\frac{T(x,t)-T_\infty}{T_i-T_\infty} = C_1 \exp\left(-\zeta_1^2 \frac{\alpha t}{L^2}\right)\cos\left(\zeta_1 x/L\right),$$

where α is the thermal diffusivity defined as $\alpha \equiv k/\rho c_p$. Coefficients C_1 and ζ_1 for this problem are given as: $C_1 = 1.1819,\ \zeta_1 = 1.2073.$