We want to design the control system for the system below

\[ G(s) = \frac{K_1}{s(s+1)} \quad \text{and} \quad H(s) = K_2s. \]

Assume that the response of the system is approximated well by a quadratic system. The design performance specifications are (i) the damping ratio of the dominant roots must be greater than \( \sqrt{3}/2 \) and (ii) the 2% settling time is less than 4 seconds.

(a) (15) Sketch the solution space (based on the assumption that the closed loop is approximated well by a second order system) generated by the settling time in (i) and the damping ratio in (ii).

(b) (15) Define the parameters \( \alpha = K_1 \) and \( \beta = K_2K_1 \). What is the characteristic equation of the closed loop system?

(c) (15) Use Routh’s array to determine (constraints that define) the range of the parameters \( \alpha, \beta \) that yield stable feedback control strategies.

(d) (20) Set \( \beta = 0 \) and sketch the root locus as \( \alpha \) varies. Choose the gain \( \alpha \) so that the poles of this root locus lie on a circle of radius \( \sqrt{2} \) about the origin in the complex plane. Does this design satisfy all the performance specifications?

(e) (20) Now fix \( \alpha = K_1 \) as in (d) and sketch the root locus as \( \beta \) varies. Choose a gain \( K_2 \) that satisfies all the performance criteria. Describe your predicted performance in terms of a solution to a second order system.

(f) (15) What is the steady state response of your design for a ramp input?